



1.6.4 Geometric interpretation of complex numbers

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Outline

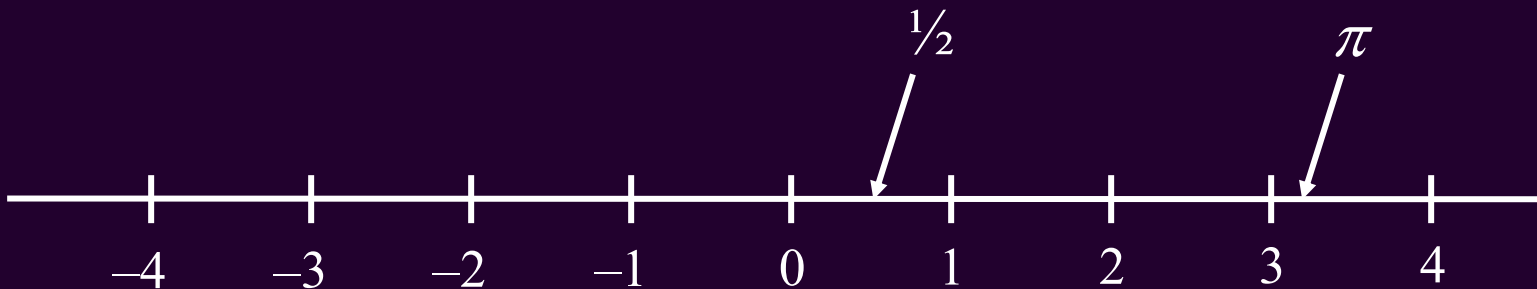
- In this topic, we will
 - Review graphs of functions and the Cartesian plane
 - Introduce the idea of the complex plane
 - Describe the real and imaginary lines and the left- and right-hand planes





Geometric interpretations

- You know how to represent:
 - Integers
 - Rational numbers
 - Real numbers



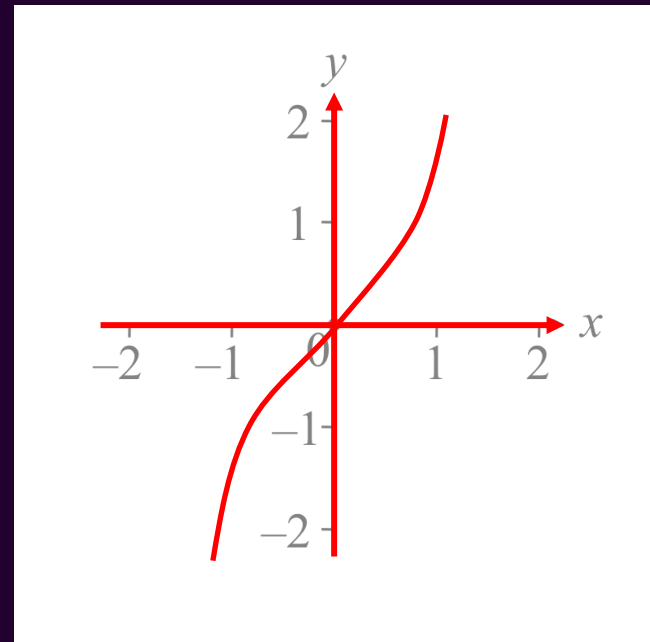


Geometric interpretations

- Question: How do we interpret $\alpha + j\beta$?
 - Both α and β are real numbers...
- Consider the representation of x and y when plotting functions

$$y = f(x)$$

- Any point where $y = 0$ is said to lie on the x -axis
- Any point where $x = 0$ is said to lie on the y -axis



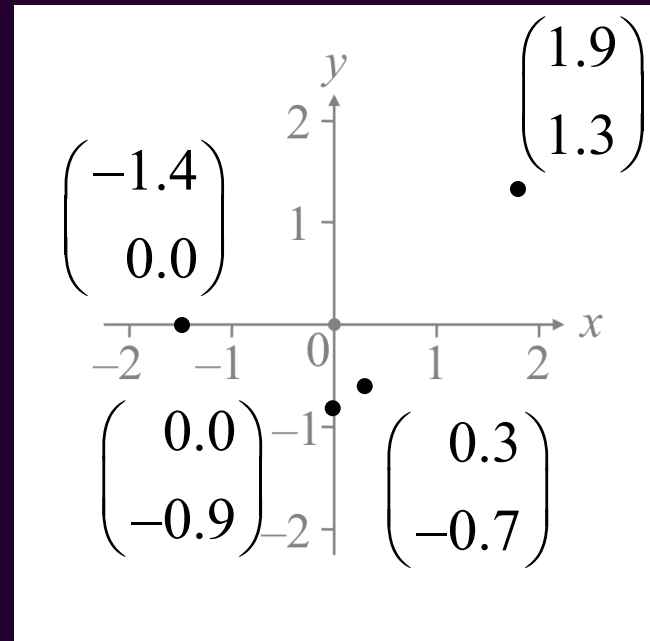


The Cartesian plane

- Question: How do we interpret $\alpha + j\beta$?
 - Both α and β are real numbers...
- Similarly, a 2-dimensional vector could be viewed on such axes

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

- Any 2D-vector where the second coordinate is zero is on the x -axis
- Any 2D-vector where the first coordinate is zero is on the y -axis
- This is called the *Cartesian plane*



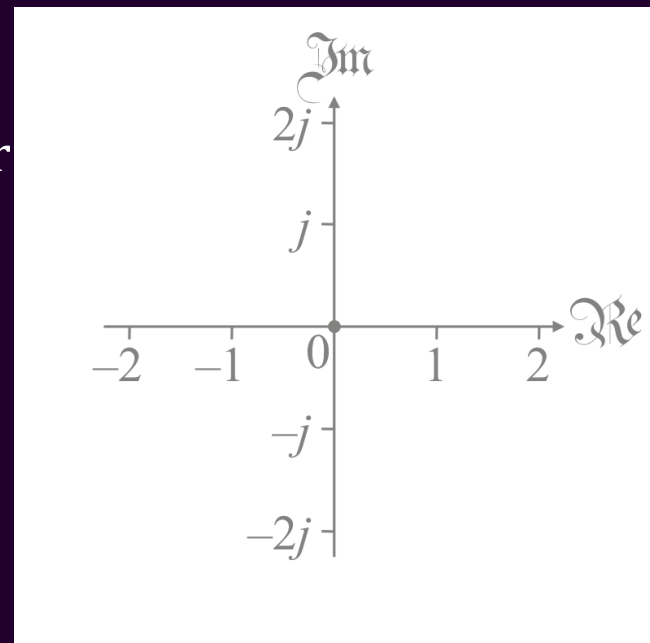


The complex plane

- We will plot the complex number $\alpha + j\beta$ in the same way as way we plot a two-dimensional vector

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

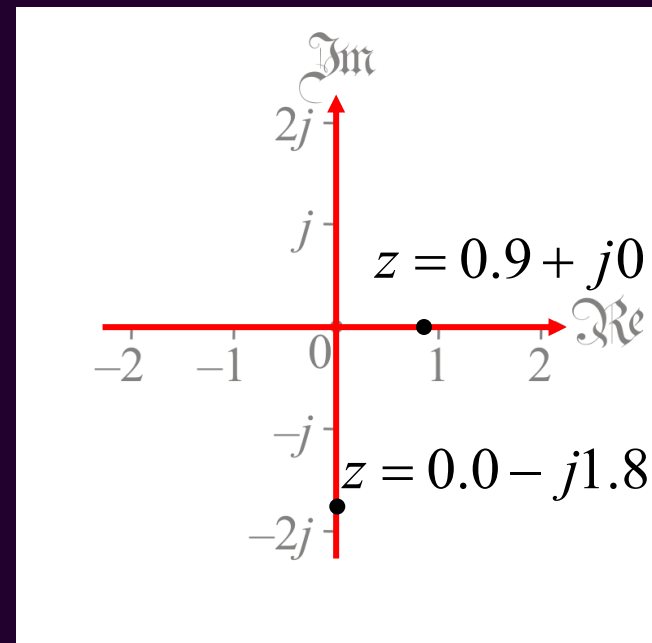
- Remember, a complex number is a single value
 - Like a real number or an integer
- It is not a two-dimensional vector
- This representation will be called the *complex plane*





Real and imaginary axes

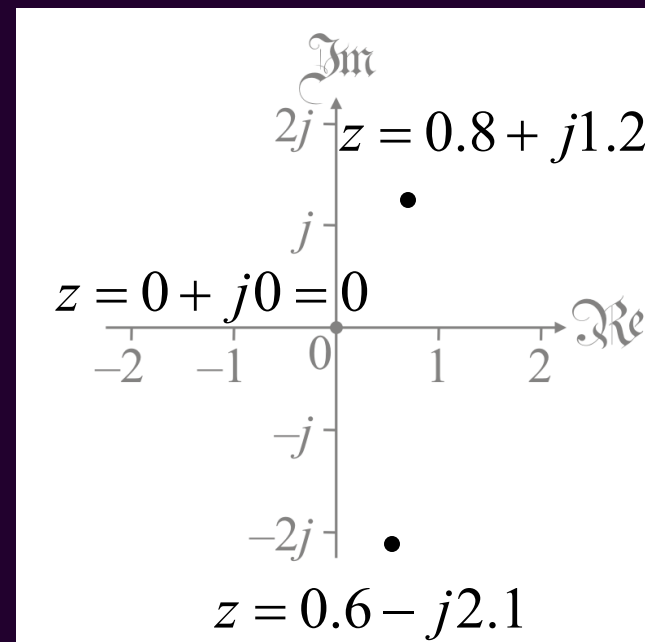
- A complex number with zero imaginary part will be said to lie on the *real axis* or the *real line*
 - If $z = 1.2 + j0$, it is still a complex number, even though it is also real
 - Just like $\frac{1}{2}$ is still a real number, even though it is also a rational
- A complex number with zero real part will be said to lie on the *imaginary axis* or *line*
 - Such numbers are called *imaginary*





The complex plane

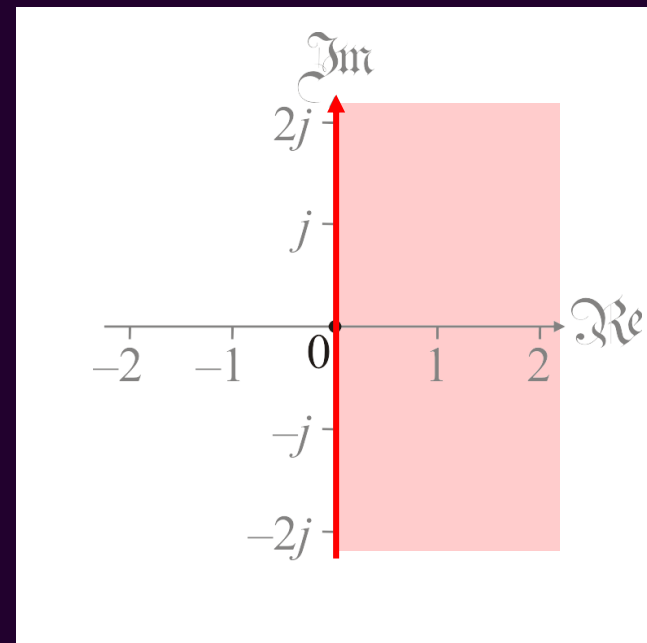
- If $\alpha + j\beta$ does not lie on either axis, we plot it so that
 - The real component lines up on the real axis
 - The imaginary component lines up on the imaginary axis





The complex plane

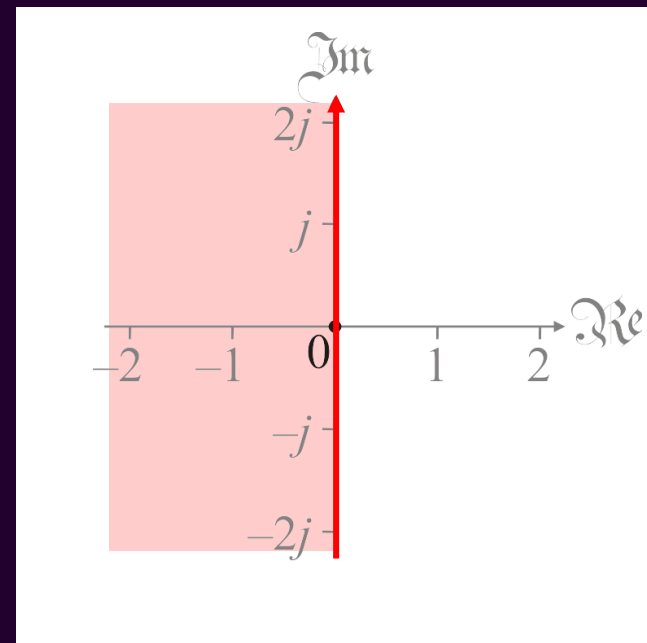
- We will classify complex numbers based on the sign of the real parts, so given $z = \alpha + j\beta$
 - If $\alpha > 0$, we will say z lies in the open right-hand plane
 - If $\alpha \geq 0$, we will say z lies in the closed right-hand plane
 - The open right-hand plane and the imaginary axis
- Recall the concept of an open interval $(0, 1)$
 - It does not contain the boundary points 0 or 1





The complex plane

- Similarly, given $z = \alpha + j\beta$
 - If $\alpha < 0$, we will say z lies in the open left-hand plane
 - If $\alpha \leq 0$, we will say z lies in the closed left-hand plane





Summary

- In this topic, you now
 - Understand the concept of the complex plane
 - Can plot a point in that plane
 - Know the terms real and imaginary axes
 - Also called the real and imaginary lines
 - Know the terms open and closed left- and right-hand planes





References

- [1] https://en.wikipedia.org/wiki/Complex_plane



Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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